I would like to take up a couple of questions on binomial probability today. The concepts of the topic have been covered in detail in the book so I am assuming that you know how to solve questions such as “What is the probability of getting at least 3 heads on 5 tosses of a coin?” etc. Therefore, let’s work on a couple of questions which use the binomial probability with a twist.

Question 1: Martin and Joey are playing a coin game in which each player tosses a fair coin alternately. The player who gets a ‘Heads’ first wins. The maximum number of tosses allowed in a single game for any player is 6. What is the probability that the person who tosses first will win the game?

Solution:

Probability of getting ‘Heads’ on a single toss = 1/2

Probability of getting ‘Tails’ on a single toss = 1/2

The person who starts the game can win the game if one of the following scenarios plays out:

1. The first person tosses the coin and gets a ‘Heads’ right away. The first person wins!
2. The first person tosses the coin and gets a ‘Tails’. The second person gets ‘Tails’ too. The first person tosses again and gets a ‘Heads’. The first person wins!
3. The first person tosses the coin and gets a ‘Tails’. The second person gets ‘Tails’ too. The first person tosses again and gets a ‘Tails’ again. The second person gets ‘Tails’ again too. Finally, the first person tosses and this time, gets a ‘Heads’. The first person wins!

and so on…

In the worst case, the first person will have to toss 6 times to get a ‘Heads’. He and the second person would end up getting ‘Tails’ on five previous tosses.

Probability that the first person tosses the coin and gets a ‘Heads’ right away = 1/2

Probability that the first person tosses the coin and gets a ‘Tails’ (1/2), the second person gets ‘Tails’ (1/2) and then the first person gets a ‘Heads’(1/2) = (1/2)*(1/2)*(1/2) = (1/2)^3

Probability that the first person tosses the coin and gets a ‘Tails’ (1/2), the second person gets ‘Tails’ (1/2) , the first person tosses again and gets a ‘Tails’ again (1/2), the second person gets ‘Tails’ again (1/2) and finally, the first person tosses and this time, gets a ‘Heads’ (1/2) = (1/2)^5

and so on…

Probability that the first person will have to toss 6 times to get a ‘Heads’ = (1/2)^11

To get the probability of the first person winning, we just need to add all these probabilities now.

Probability that the first person will win = (1/2) + (1/2)^3 + (1/2)^5 + (1/2)^7 + (1/2)^9 + (1/2)^11

On the same lines, can you find the probability that the person who tosses second wins? I hope you understand that it is very similar to what we have already discussed. The person who tosses second will win if one of the following happens:

1. The person who tosses first gets ‘Tails’ and then the person who tosses second gets ‘Heads’.
2. The person who tosses first gets ‘Tails’, the person who tosses second gets ‘Tails’, the person who tosses first gets ‘Tails’ again and the second person then gets ‘Heads’.

and so on…

Probability that the second person will win = (1/2)^2 + (1/2)^4 + (1/2)^6 + (1/2)^8 + (1/2)^10 + (1/2)^12

I hope you see that the question is quite straight forward. Now, let's take a question very similar to one from a GMAT Prep test.

Question 2: For one toss of a certain coin, the probability that the outcome is heads is 0.7. If the coin is tossed 6 times, what is the probability that the outcome will be tails at least 5 times?

Solution: This question is very similar to the questions we saw in the Probability book. The only difference is that we are not tossing a fair coin. The probability of getting heads is 0.7 not 0.5. So the probability of getting tails must be 0.3 since the total probability has to add up to 1.

The only acceptable cases are those in which we get 'tails' on all 6 tosses or we get tails on exactly 5 of the 6 tosses.

P(Tails on all 6 tosses) = (0.3)*(0.3)*(0.3)*(0.3)*(0.3)*(0.3) = (0.3)^6

P(Tails on exactly 5 tosses and Heads on one toss) = (0.3)^5*(0.7)*6

We multiply by 6 because 5 tails and 1 heads can be obtained in 6 different ways: HTTTTT, THTTTT, TTHTTT, TTTHTT, TTTTHT, TTTTTH

Probability that the outcome will be tails at least 5 times = Probability that the outcome will be tails 5 times + Probability that the outcome will be tails 6 times

Probability that the outcome will be tails at least 5 times = (0.3)^6 + (0.3)^5*(0.7)*6

Again, the question is straight forward. It just has a little twist which sometimes throws people off during the test. It is these little things that differentiate a medium level question from a high level question.